

Hong Kong Mathematics Olympiad (2005 – 2006)

Heat Event (Individual)

香港数学竞赛 (2005 – 2006)

初赛项目(个人)

除非特别声明，答案须用数字表达，并化至最简。

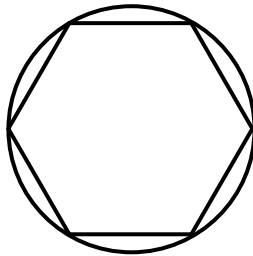
Unless otherwise stated, all answers should be expressed in numerals in their simplest form.

1. 设 $\sqrt{20 + \sqrt{300}} = \sqrt{x} + \sqrt{y}$ 及 $w = x^2 + y^2$ ，求 w 的值。

Let $\sqrt{20 + \sqrt{300}} = \sqrt{x} + \sqrt{y}$ and $w = x^2 + y^2$, find the value of w .

2. 如图一，一个正六边形内接于一个圆周长为 4 m 的圆内。设该正六边形的面积是 $A \text{ m}^2$ ，求 A 的值。(取 $\pi = \frac{22}{7}$)

In Figure 1, a regular hexagon is inscribed in a circle with circumference 4 m. If the area of the regular hexagon is $A \text{ m}^2$, find the value of A . (Take $\pi = \frac{22}{7}$)



图一

Figure 1

3. 已知 $\frac{1}{2 + \frac{3}{1 + \frac{1}{x}}} = \frac{5}{28}$ ，求 x 的值。

Given that $\frac{1}{2 + \frac{3}{1 + \frac{1}{x}}} = \frac{5}{28}$, find the value of x .

4. 设 $A = \frac{2006}{20052005^2 - 20052004 \times 20052006}$ ，求 A 的值。

Let $A = \frac{2006}{20052005^2 - 20052004 \times 20052006}$, find the value of A .

5. 已知 $4\sec^2\theta - \tan^2\theta - 7\sec\theta + 1 = 0$ 及 $0^\circ \leq \theta \leq 180^\circ$ ，求 θ 的值。

Given that $4\sec^2\theta - \tan^2\theta - 7\sec\theta + 1 = 0$ and $0^\circ \leq \theta \leq 180^\circ$, find the value of θ .

6. 已知 w 、 x 、 y 和 z 是正整数且满足方程 $w + x + y + z = 12$ 。若方程有 W 组不同的正整数解，求 W 的值。

Given that w, x, y and z are positive integers which satisfy the equation $w + x + y + z = 12$. If there are W sets of different positive integral solutions of the equation, find the value of W .

7. 已知在数列 $1001, 1001001, 1001001001, \dots, \underbrace{1001}_{2}\underbrace{001}_{2}\dots\underbrace{1001}_{2}, \dots$ 中有 R 个质数，求 R 的值。

Given that the number of prime numbers in the sequence $1001, 1001001, 1001001001, \dots, \underbrace{1001}_{2}\underbrace{001}_{2}\dots\underbrace{1001}_{2}, \dots$ is R , find the value of R .

8. 设 $\lfloor x \rfloor$ 表示不大于 x 的最大整数，例如 $\lfloor 2.5 \rfloor = 2$ 。若 $B = \left\lceil \log_7 \left(462 + \log_2 \lfloor \tan 60^\circ \rfloor + \sqrt{9872} \right) \right\rceil$ ，求 B 的值。

Let $\lfloor x \rfloor$ be the largest integer not greater than x , for example, $\lfloor 2.5 \rfloor = 2$. If

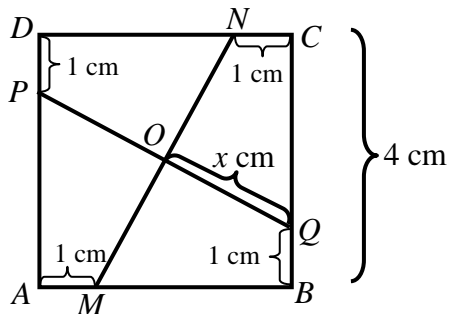
$B = \left\lceil \log_7 \left(462 + \log_2 \lfloor \tan 60^\circ \rfloor + \sqrt{9872} \right) \right\rceil$, find the value of B .

9. 已知 7^{2006} 的个位数是 C ，求 C 的值。

Given that the units digit of 7^{2006} is C , find the value of C .

10. 如图二， $ABCD$ 是一正方形，其边长为 4 cm。线段 PQ 和 MN 相交于点 O 。若 PD 、 NC 、 BQ 和 AM 的长度是 1 cm， OQ 的长度是 x cm，求 x 的值。

In Figure 2, $ABCD$ is a square with side length equal to 4 cm. The line segments PQ and MN intersect at the point O . If the lengths of PD , NC , BQ and AM are 1 cm and the length of OQ is x cm, find the value of x .



图二

Figure 2